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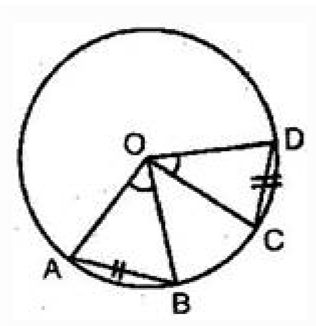
Chapter 9: Circles - NCERT Solutions for Class 9 Maths | Expert Answers & PDF

Ex 9.1 Question 1.

Recall that two circles are congruent if they have the same radii. Prove that equal chords of congruent circles subtend equal angles at their centres.

Answer.

I Part: Two circles are said to be congruent if and only if one of them can be superposed on the other so as to cover it exactly.



Let $\mathrm{C}(\mathrm{O}, r)$ and $\mathrm{C}\left(\mathrm{O}^{\prime}, s\right)$ be two circles. Let us imagine that the circle $\mathrm{C}\left(\mathrm{O}^{\prime}, s\right)$ is superposed on $\mathrm{C}(\mathrm{O}, r)$ so that $O^{\prime}$ coincide with O. Then it can easily be seen that $C\left(O^{\prime}, s\right)$ will cover $C(O, r)$ completely if and only if $r=s$.

Hence we can say that two circles are congruent, if and only if they have equal radii.

II Part: Given: In a circle (O, r), AB and CD are two equal chords, subtend $\angle \mathrm{AOB}$ and $\angle \mathrm{COB}$ at the centre.

To Prove: $\angle \mathrm{AOB}=\angle \mathrm{COD}$

Proof: In $\triangle \mathrm{AOB}$ and $\triangle COD$,
$AB=CD[$ Given $]$
$AO=CO[$ Radii of the same circle]
$\mathrm{BO}=\mathrm{DO}$ [Radii of the same circle]
$\therefore \triangle \mathrm{AOB} \cong \triangle$ COD [By SSS axiom]
$\Rightarrow \angle \mathrm{AOB}=\angle \mathrm{COD}[\mathrm{By} \quad \mathrm{CPCT}]$

Hence Proved.

Ex 9.1 Question 2.

Prove that if chords of congruent circles subtend equal angles at their centres, then the chords are equal.

Answer.

Given: In a circle $(\mathrm{O}, r), \mathrm{AB}$ and CD subtend two angles at the centre such that $\angle \mathrm{AOB}=\angle \mathrm{COD}$

To Prove: $AB=CD$

Proof: : In $\triangle AOB$ and $\triangle COD$,
$\mathrm{AO}=\mathrm{CO}$ [Radii of the same circle]
$\mathrm{BO}=\mathrm{DO}$ [Radii of the same circle]
$\angle \mathrm{AOB}=\angle \mathrm{COD}$ [Given]
$\therefore \triangle \mathrm{AOB} \cong \triangle COD$ [By SAS axiom]
$\Rightarrow \mathrm{AB}=\mathrm{CD}[\mathrm{By}$ CPCT]

Hence proved.

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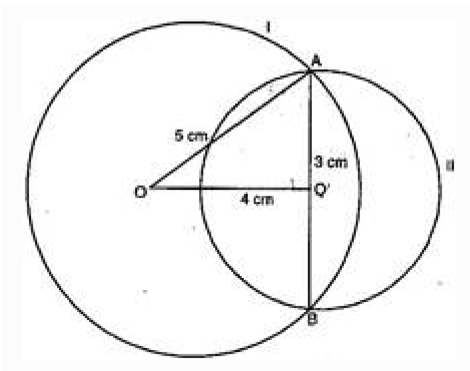
Ex 9.2 Question 1.

Two circles of radii $5 \mathrm{~cm}$ and $3 \mathrm{~cm}$ intersect at two points and the distance between their centers is $4 \mathrm{~cm}$. Find the length of the common chord.

Answer.

Let two circles with centres O and O ' intersect each other at points A and B. On joining A and B, AB is a common chord.



Radius $\mathrm{OA}=5 \mathrm{~cm}$, Radius $\mathrm{O}^{\prime} \mathrm{A}=3 \mathrm{~cm}$,

Distance between their centers $O \mathrm{O}^{\prime}=4 \mathrm{~cm}$

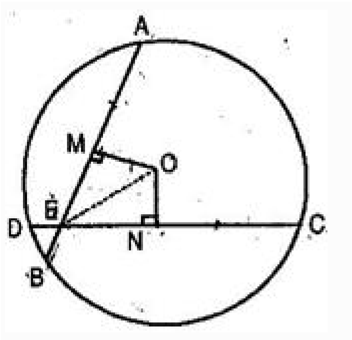
In triangle AOO,
$5^2=4^2+3^2$
$\Rightarrow 25=16+9$
$\Rightarrow 25=25$

Hence $AOO^{\prime}$ is a right triangle, right angled at $O^{\prime}$.

Since, perpendicular drawn from the center of the circle bisects the chord.

Hence $\mathrm{O}^{\prime}$ is the mid-point of the chord AB. Also $\mathrm{O}^{\prime}$ is the centre of the circle II.

Therefore length of chord $AB=$ Diameter of circle II
$\therefore$ Length of chord $\mathrm{AB}=2 \times 3=6 \mathrm{~cm}$.



To prove: (a) $AE=CE$
(b) $BE=DE$

Construction: Draw $\mathrm{OM} \perp \mathrm{AB}, \mathrm{ON} \perp \mathrm{CD}$. Also join OE.

Proof: In right triangles OME and ONE,
$\angle \mathrm{OME}=\angle \mathrm{ONE}=90^{\circ}$
$\mathrm{OM}=\mathrm{ON}$
[Equal chords are equidistance from the centre]
$\mathrm{OE}=\mathrm{OE}[$ Common]
$\therefore \triangle \mathrm{OME} \cong \triangle$ ONE [RHS rule of congruency]
$\therefore ME=NE[ByCPCT]$

Now, O is the centre of circle and $\mathrm{OM} \perp \mathrm{AB}$
$\therefore AM=\frac{1}{2}AB$
[Perpendicular from the centre bisects the chord]
Similarly, $\mathrm{NC}=\frac{1}{2}\mathrm{CD}$.(iii)

But $AB=CD$ [Given]

From eq. (ii) and (iii), $\mathrm{AM}=\mathrm{NC}$ (iv)

Also $\mathrm{MB}=\mathrm{DN}$ (V)

Adding (i) and (iv), we get,
$AM+ME=NC+NE$
$\Rightarrow \mathrm{AE}=\mathrm{CE}$ [Proved part (a)]

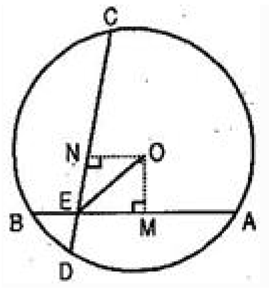
Now $AB=CD$ [Given]
$\mathrm{AE}=\mathrm{CE}[$ Proved $]$
$\Rightarrow AB-AE=CD-CE$
$\Rightarrow BE=DE$ [Proved part (b)]

Ex 9.2 Question 3.

If two equal chords of a circle intersect within the circle, prove that the line joining the point of intersection to the centre makes equal angles with the chord.

Answer.

Given: AB and CD be two equal chords of a circle with centre O intersecting each other with in the circle at point E. OE is joined.



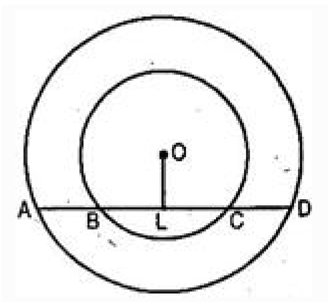
To prove: $\angle \mathrm{OEM}=\angle \mathrm{OEN}$

Construction: Draw $\mathrm{OM} \perp \mathrm{AB}$ and
$ON \perp CD$.

Proof: In right angled triangles OME and ONE,
$\angle \mathrm{OME}=\angle \mathrm{ONE}[$ Each 90° $]$
$\mathrm{OM}=\mathrm{ON}$ [Equal chords are equidistant from the centre]
$OE=OE[$ Common $]$
$\therefore \triangle \mathrm{OME} \cong \triangle \mathrm{ONE}[RHS$ rule of congruency]
$\therefore \angle \mathrm{OEM}=\angle \mathrm{OEN}$ [By CPCT]

Ex 9.2 Question 4.

If a line intersects two concentric circles (circles with the same centre) with centre O at A, B, C and D, prove that $AB=CD$. (See figure)



Answer.
Given: Line l intersects two concentric circles with centre O at points A, B, C and D.

To prove: $AB=CD$

Construction: Draw $\mathrm{OL} \perp l$

Proof: AD is a chord of outer circle and $\mathrm{OL} \perp \mathrm{AD}$.
$\therefore AL=LD$.(i) [Perpendicular drawn from the centre bisects the chord]

Now, BC is a chord of inner circle and
$\mathrm{OL} \perp \mathrm{BC}$
$\therefore BL = LC$...(ii) [Perpendicular drawn from the centre bisects the chord]

Subtracting (ii) from (i), we get,
$AL - BL = LD - LC$
$\Rightarrow AB = CD$

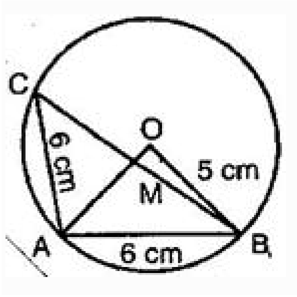
Ex 9.2 Question 5.

Three girls Reshma, Salma and Mandip are standing on a circle of radius $\mathbf{5\ m}$ drawn in a park. Reshma throws a ball to Salma, Salma to Mandip, Mandip to Reshma. If the distance between Reshma and Salma and between Salma and Mandip is $6\ \mathrm{m}$ each, what is the distance between Reshma and Mandip?

Answer.

Let Reshma, Salma and Mandip takes the position C, A and B on the circle. Since $AB = AC$

The centre lies on the bisector of $\angle \mathrm{BAC}$.



Let M be the point of intersection of BC and OA.

Again, since $AB = AC$ and AM bisects
$\angle \mathrm{CAB}$.
$\therefore AM \perp CB$ and M is the mid-point of CB.

Let $OM = x$, then $MA = 5 - x$

From right angled triangle OMB,
$OB^2 = OM^2 + MB^2$
$\Rightarrow 5^2 = x^2 + \mathrm{MB}^2$

Again, in right angled triangle AMB,
$AB^2 = AM^2 + MB^2$
$\Rightarrow 6^2 = (5 - x)^2 + MB^2$.

Equating the value of MB^2 from eq. (i) and (ii),
$5^2 - x^2 = 6^2 - (5 - x)^2$
$\Rightarrow (5 - x)^2 - x^2 = 6^2 - 5^2$
$\Rightarrow 25 - 10x + x^2 - x^2 = 36 - 25$
$\Rightarrow 10x = 25 - 11$
$\Rightarrow 10x = 14$
$\Rightarrow x = \frac{14}{10}$

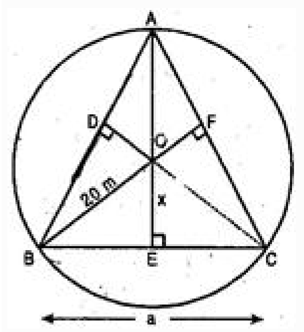
Hence, from eq. (i),
$MB^2 = 5^2 - x^2 = 5^2 - \left(\frac{14}{10}\right)^2$
$= \left(5 + \frac{4}{10}\right)\left(5 - \frac{14}{10}\right) = \frac{64}{10} \times \frac{36}{10}$
$\Rightarrow MB = \frac{8 \times 6}{10} = 4.8\ \mathrm{cm}$
$\therefore BC = 2MB = 2 \times 4.8 = 9.6\ \mathrm{cm}$

Ex 9.2 Question 6.

A circular park of radius $\mathbf{20}\mathrm{m}$ is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary each having a toy telephone in his hands to talk each other. Find the length of the string of each phone.

Answer.

Let position of three boys Ankur, Syed and David are denoted by the points A, B and C respectively.



$A = B = C = a$[say]

Since equal sides of equilateral triangle are as equal chords and perpendicular distances of equal chords of a circle are equidistant from the centre.

$\therefore OD = OE = OF = x$ cm[say]

Join OA, OB and OC.

$\Rightarrow$ Area of $\triangle AOB$

$=$ Area of $\triangle BOC =$ Area of $\triangle AOC$

And Area of $\triangle ABC$

$=$ Area of $\triangle AOB +$ Area of $\triangle BOC +$ Area of $\triangle AOC$

$\Rightarrow$ And Area of $\triangle ABC = 3\times$ Area of $\triangle BOC$

$$\Rightarrow \frac{\sqrt{3}}{4} a^2 = 3\left(\frac{1}{2} BC \times OE\right)$$

$$\Rightarrow \frac{\sqrt{3}}{4} a^2 = 3\left(\frac{1}{2} \times a \times x\right)$$

$$\Rightarrow \frac{a^2}{a} = 3 \times \frac{1}{2} \times \frac{4}{\sqrt{3}} \times x$$

$$\Rightarrow a = 2\sqrt{3} x \ldots\ldots\ldots \text{(i)}$$

Now, $CE \perp BC$

$\therefore BE = EC = \frac{1}{2} BC$ [$\because$ Perpendicular drawn from the centre bisects the chord]

$\Rightarrow BE = EC = \sqrt{3} x$

Now in right angled triangle BEO,

$OE^2 + BE^2 = OB^2$ [Using Pythagoras theorem]

$\Rightarrow x^2 + (\sqrt{3} x)^2 = (20)^2$

$$\Rightarrow x^2 + 3x^2 = 400$$

$$\Rightarrow 4x^2 = 400$$

$$\Rightarrow x^2 = 100$$

$$\Rightarrow x = 10 \text{ m}$$

And $a = 2\sqrt{3} x = 2\sqrt{3} \times 10 = 20\sqrt{3}$ m

Thus distance between any two boys is $20\sqrt{3}$ m.

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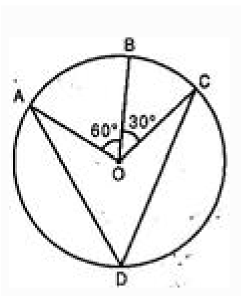
Ex 9.3 Question 1.

In figure, $\mathbf{A}, \mathbf{B}, \mathbf{C}$ are three points on a circle with centre O such that $\angle \mathbf{BOC} = 30^{\circ}, \angle \mathbf{AOB} = 60^{\circ}$. If D is a point on the circle other than the arc ABC, find $\angle \mathrm{ADC}$.



Answer.

$\angle \mathrm{AOC} = \angle \mathrm{AOB} + \angle \mathrm{BOC}$
$\Rightarrow \angle \mathrm{AOC} = 60^{\circ} + 30^{\circ} = 90^{\circ}$

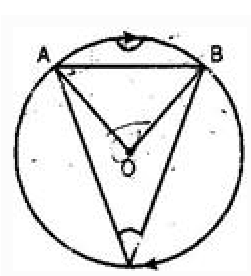
Now $\angle \mathrm{AOC} = 2 \angle \mathrm{ADC}$
[$\because$ Angled subtended by an arc, at the centre of the circle is double the angle subtended by the same arc at any point in the remaining part of the circle]
$\Rightarrow \angle \mathrm{ADC} = \frac{1}{2} \angle \mathrm{AOC}$
$\Rightarrow \angle \mathrm{ADC} = \frac{1}{2} \times 90^{\circ} = 45^{\circ}$

Ex 9.3 Question 2.

A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord on a point on the minor arc and also at a point on the major arc.

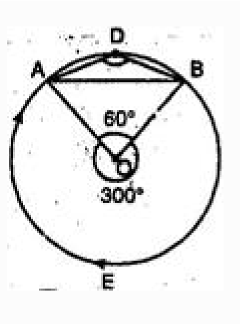
Answer.

Let AB be the minor arc of circle.



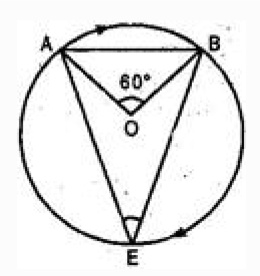
$\therefore$ Chord $\mathrm{AB} =$ Radius $\mathrm{OA} =$ Radius OB
$\therefore \triangle \mathrm{AOB}$ is an equilateral triangle.
$\Rightarrow \angle \mathrm{AOB} = 60^{\circ}$

Now $m\widehat{\mathrm{AB}} + m\widehat{\mathrm{BA}} = 360^{\circ}$
$\Rightarrow \angle \mathrm{AOB} + \angle \mathrm{BOA} = 360^{\circ}$
$\Rightarrow 60^{\circ} + \angle \mathrm{BOA} = 360^{\circ}$



$\Rightarrow \angle \mathrm{BOA}=360^{\circ}-60^{\circ}=300^{\circ}$
D is a point in the minor arc.

$\therefore m\overparen{\mathrm{BA}}=2 \angle \mathrm{BDA}$
$\Rightarrow \angle \mathrm{BOA}=2 \angle \mathrm{BDA}$
$\Rightarrow \angle \mathrm{BDA}=\frac{1}{2} \angle \mathrm{BOA}=\frac{1}{2} \times 300^{\circ}$
$\Rightarrow \angle \mathrm{BDA}=150^{\circ}$

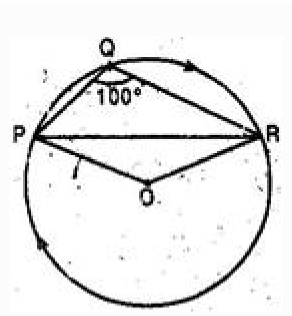


Thus angle subtended by major arc, $\overparen{\mathrm{BA}}$ at any point D in the minor arc is 150°.

Let E be a point in the major arc $\overparen{\mathrm{BA}}$.
$\therefore m\overparen{\mathrm{AB}}=2 \angle \mathrm{AEB}$
$\Rightarrow \angle \mathrm{AOB}=2 \angle \mathrm{AEB}$
$\Rightarrow \angle \mathrm{AEB}=\frac{1}{2} \angle \mathrm{AOB}$
$\Rightarrow \angle \mathrm{AEB}=\frac{1}{2} \times 60^{\circ}=30^{\circ}$

Ex 9.3 Question 3.

In figure, $\angle \mathbf{PQR}=100^{\circ}$: where $\mathbf{P}, \mathbf{Q}, \mathbf{R}$ are points on a circle with centre $\mathbf{O}$. Find $\angle \mathbf{OPR}$.



Answer.

In the figure, Q is a point in the minor arc $\overline{\mathrm{PQR}}$.

$\therefore m\overparen{RP}=2 \angle \mathrm{PQR}$
$\Rightarrow \angle \mathrm{ROP}=2 \angle \mathrm{PQR}$
$\Rightarrow \angle \mathrm{ROP}=2 \times 100^{\circ}=200^{\circ}$

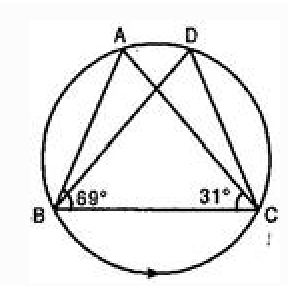
Now $m\overparen{\mathrm{PR}}+m\overparen{\mathrm{RP}}=360^{\circ}$
$\Rightarrow \angle \mathrm{POR}+\angle \mathrm{ROP}=360^{\circ}$
$\Rightarrow \angle \mathrm{POR}+200^{\circ}=360^{\circ}$
$\Rightarrow \angle \mathrm{POR}=360^{\circ}-200^{\circ}=160^{\circ} \ldots \ldots$ (i)

Now $\triangle \mathrm{OPR}$ is an isosceles triangle.
$\therefore \mathrm{OP}=\mathrm{OR}$ [radii of the circle]
$\Rightarrow \angle \mathrm{OPR}=\angle \mathrm{ORP}$ [angles opposite to equal sides are equal]

Now in isosceles triangle OPR,
$\angle \mathrm{OPR}+\angle \mathrm{ORP}+\angle \mathrm{POR}=180^{\circ}$
$\Rightarrow \angle \mathrm{OPR}+\angle \mathrm{ORP}+160^{\circ}=180^{\circ}$
$\Rightarrow 2 \angle \mathrm{OPR}=180^{\circ}-160^{\circ}$ [Using (i) \& (ii)]
$\Rightarrow 2 \angle \mathrm{OPR}=20^{\circ}$
$\Rightarrow \angle \mathrm{OPR}=10^{\circ}$

Ex 9.3 Question 4.

In figure, $\angle \mathbf{ABC}=69^{\circ}, \angle \mathbf{ACB}=31^{\circ}$: find $\angle \mathbf{BDC}$.



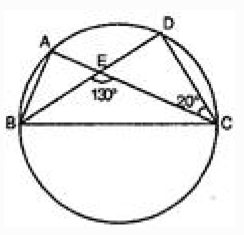
Answer.

In triangle ABC,
$\angle\mathrm{BAC}+\angle\mathrm{ABC}+\angle\mathrm{ACB}=180^{\circ}$
$\Rightarrow\angle\mathrm{BAC}+69^{\circ}+31^{\circ}=180^{\circ}$
$\Rightarrow\angle\mathrm{BAC}=180^{\circ}-69^{\circ}-31^{\circ}$
$\Rightarrow\angle\mathrm{BAC}=80^{\circ}\ldots\ldots$ (i)

Since, A and D are the points in the same segment of the circle.
$\therefore\angle\mathrm{BDC}=\angle\mathrm{BAC}$
[Angles subtended by the same arc at any points in the alternate segment of a circle are equal] $\Rightarrow\angle\mathrm{BDC}=80^{\circ}$[Using (i)]

Ex 9.3 Question 5.

In figure, A, B, C, D are four points on a circle. AC and BD intersect at a point E such that $\angle$ BEC $=130^{\circ}$ and $\angle ECD=20^{\circ}$. Find $\angle$ BAC.



Answer.

Given: $\angle\mathrm{BEC}=130^{\circ}$ and $\angle\mathrm{ECD}=20^{\circ}$
$\angle\mathrm{DEC}=180^{\circ}-\angle\mathrm{BEC}=180^{\circ}-130^{\circ}=50^{\circ}$ [Linear pair]

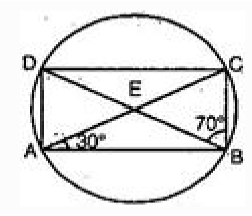
Now in $\triangle\mathrm{DEC}$,
$\angle\mathrm{DEC}+\angle\mathrm{DCE}+\angle\mathrm{EDC}=180^{\circ}$ [Angle sum property]
$\Rightarrow 50^{\circ}+20^{\circ}+\angle EDC=180^{\circ}$
$\Rightarrow\angle\mathrm{EDC}=110^{\circ}$
$\Rightarrow\angle\mathrm{BAC}=\angle\mathrm{EDC}=110^{\circ}$ [Angles in same segment]

Ex 9.3 Question 6.

$\mathbf{ABCD}$ is a cyclic quadrilateral whose diagonals intersect at a point E. $\angle\mathrm{DBC}={}^{70^{\circ}}=\angle\mathrm{BAC}$ is 30° find $\angle BCD$. Further if $AB=BC$, find $\angle ECD$.

Answer.

For chord CD



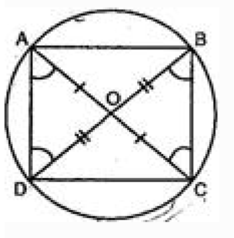
$\angle BCD=80^{\circ}\angle CBD=\angle CAD$ (Angles in same segment)
$\angle CAD=70^{\circ}$
$\angle BAD=\angle BAC+\angle CAD$
$=30^{\circ}+70^{\circ}=100^{\circ}$
$\angle BCD+\angle BAD=180^{\circ}$ (Opposite angles of a cyclic quadrilateral)
$\angle BCD+100=180^{\circ}$
$\angle BCD=80^{\circ}$
In $\triangle ABC$
$\mathrm{AB}=\mathrm{BC}$ (given)
$\therefore\angle BCA=\angle CAB$ (Angles opposite to equal sides of a triangle)
$\angle BCA=30^{\circ}$
We have $\angle BCD=80^{\circ}\angle BCA+\angle ACD=80^{\circ}$
$30^{\circ}+\angle ACD=80^{\circ}$
$\angle ACD=50^{\circ}$
$\angle ECD=50^{\circ}$

Ex 9.3 Question 7.

If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of the quadrilateral, prove that it is a rectangle.

Answer.

Let ABCD a cyclic quadrilateral having diagonals as BD and AC intersecting each other at point O.



$\angle BAD = \frac{1}{2}\angle BOD = \frac{180^\circ}{2} = 90^\circ$
(Consider BD as a chord) $\angle BCD + \angle BAD = 180^\circ$ (Cyclic quadrilateral)
$\angle BCD = 180^\circ - 90^\circ = 90^\circ \angle ADC = \frac{1}{2}\angle AOC = \frac{180^\circ}{2} = 90^\circ$
(Considering AC as a chord)
$\angle ADC + \angle ABC = 180^\circ$ (Cyclic quadrilateral)
$90^\circ + \angle ABC = 180^\circ$
$\angle ABC = 90^\circ$

Here, each interior angle of cyclic quadrilateral is of 90°. Hence it is a rectangle.

Ex 9.3 Question 8.

If the non-parallel sides of a trapezium are equal, prove that it is cyclic.

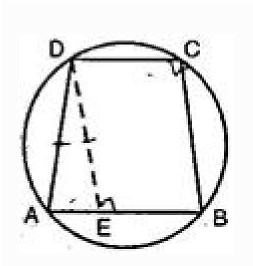
Answer.

Given: A trapezium $ABCD$ in which $AB^{\|}CD$ and $AD = BC$.

To prove: The points $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}$ are concyclic.

Construction: Draw DE $^{\|}\mathrm{CB}$.

Proof: Since $\mathrm{DE}^{\|}{}_{\mathrm{CB}}$ and $\mathrm{EB}\|_{\mathrm{DC}}$.



$\therefore EBCD$ is a parallelogram.
$\therefore \mathrm{DE} = \mathrm{CB}$ and $\angle\mathrm{DEB} = \angle\mathrm{DCB}$

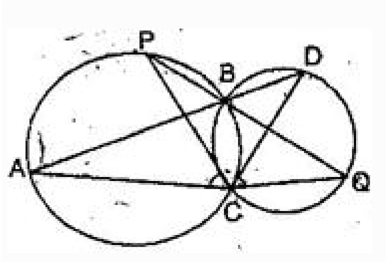
Now $\mathrm{AD} = \mathrm{BC}$ and $\mathrm{DA} = \mathrm{DE}$
$\Rightarrow \angle\mathrm{DAE} = \angle\mathrm{DEB}$

But $\angle\mathrm{DEA} + \angle\mathrm{DEB} = 180^\circ$
$\Rightarrow \angle\mathrm{DAE} + \angle\mathrm{DCB} = 180^\circ$
$[\because \angle\mathrm{DEA} = \angle\mathrm{DAE}$ and $\angle\mathrm{DEB} = \angle\mathrm{DCB}]$
$\Rightarrow \angle\mathrm{DAB} + \angle\mathrm{DCB} = 180^\circ$
$\Rightarrow \angle\mathrm{A} + \angle\mathrm{C} = 180^\circ$

Hence, $ABCD$ is a cyclic trapezium.

Ex 9.3 Question 9.

Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D, P, Q respectively (see figure). Prove that $\angle\mathrm{ACP} = \angle$ QCD.



Answer.

In triangles ACD and QCP,
$\angle\mathrm{A} = \angle\mathrm{P}$ and $\angle\mathrm{Q} = \angle\mathrm{D}$ [Angles in same segment]
$\therefore \angle\mathrm{ACD} = \angle\mathrm{QCP}$ [Third angles] ....(i)

Subtracting $\angle\mathrm{PCD}$ from both the sides of eq. (i), we get,
$\angle\mathrm{ACD} - \angle\mathrm{PCD} = \angle\mathrm{QCP} - \angle\mathrm{PCD}$
$\Rightarrow \angle\mathrm{ACPO} = \angle\mathrm{QCD}$

Hence proved.

Ex 9.3 Question 10.

If circles are drawn taking two sides of a triangle as diameters, prove that the point of intersection of these circles lie on the third side.

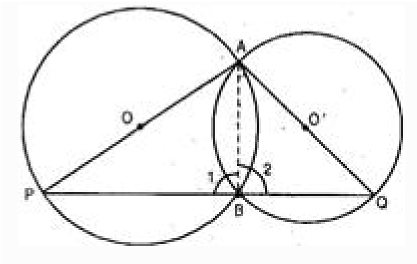
Answer.

Given: Two circles intersect each other at points A and B. AP and AQ be their respective diameters.

To prove: Point B lies on the third side PQ.

Construction: Join A and B.

Proof: AP is a diameter.



$\therefore \angle 1=90^{\circ}$
[Angle in semicircle]
AlsoAQ is a diameter.
$\therefore \angle 2=90^{\circ}$
[Angle in semicircle]
$\angle 1+\angle 2=90^{\circ}+90^{\circ}$
$\Rightarrow \angle \mathrm{PBQ}=180^{\circ}$
$\Rightarrow PBQ$ is a line.

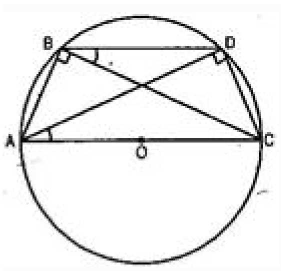
Thus point B. i.e. point of intersection of these circles lies on the third side i.e., on PQ.

Ex 9.3 Question 11.

ABC and ADC are two right triangles with common hypotenuse AC. Prove that $\angle$ CAD $=\angle$ ABD.

Answer.

We have ABC and ADC two right triangles, right angled at B and D respectively.



$\Rightarrow \angle \mathrm{ABC}=\mathrm{ADC}\,[\text{ Each } 90^{\circ}]$

If we draw a circle with AC (the common hypotenuse) as diameter, this circle will definitely passes through of an arc AC, Because B and D are the points in the alternate segment of an arc AC.

Now we have $\overparen{\mathrm{CD}}$ subtending $\angle \mathrm{CBD}$ and $\angle \mathrm{CAD}$ in the same segment.
$\therefore \angle CAD=\angle CBD$

Hence proved.